Johnston-Berry-Mielke E (es_jbm_e)

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Introduction

The *es_jbm_e* function calculates the effect size Johnston-Berry-Mielke E.

This document contains the details on how to use the functions, and formulas used in them.

1 About the Function

1.1 Input parameters:

- **chi2**
  the chi-square test statistic.

- **n**
  the sample size

- **minExp**
  the minimum expected count of the expected counts for the different categories

- *Optional parameters*
  - **test** (default is "chi") Either:
    - "chi": for chi-square based tests
    - "likelihood": for likelihood ratio tests

1.2 Output

- **Value**
  The Johnston-Berry-Mielke E value

- **Classification**
  A classification of the value by converting it to Cohen's w. This only is shown for chi-square based tests. With Excel this is only used in the array function

1.3 Dependencies

- **Excel**
  None.
  You can run the **es_jbm_e_addHelp** macro so that the function will be available with some help in the 'User Defined' category in the functions overview.

- **Python**
  The *log* function from Python's *math* library is used.

- **R**
  No other libraries required.

2 Examples

2.1 Excel

	A	B	C	D	E	F
1						
2						
3		chi-square value:		3,105263		
4		sample size:		19		
5		min. expected count:		4,75		
6						
7		test	JBM E			
8		chi	0,0544783	*=es_jbm_e(D3;D4;D5;B8)*		
9		likelihood	0,05894668	*=es_jbm_e(D3;D4;D5;B9)*		
10						
11		*JBM E*	*Qualification*			
12		*0,0544783*	*medium*			
13						
14		B11:C12	=es_jbm_e_arr(D3;D4;D5)			
15						

2.2 Python

```
[2]: chi2Value = 3.105263
     n = 19
     minExp = n/4

     es_jbm_e(chi2Value, n, minExp)
```

[2]:

	JBM E	Classification
0	0.054478	medium

```
[3]: es_jbm_e(chi2Value, n, minExp, test="likelihood")
```

```
[3]: 0.05894667803626296
```

2.3 R

```
> chi2Value <- 3.105263
> n <- 19
> minExp <- n/4
> es_jbm_e(chi2Value, n, minExp)
          E   qual
1 0.0544783 medium
> es_jbm_e(chi2Value, n, minExp, test="likelihood")
[1] 0.05894668
```

3 Details of Calculations

3.1 The Effect Size

For a chi-square:

$$E_{\chi^2_{GoF}} = \frac{q}{1-q} \times \left(\left(\sum_{i=1}^{k} \frac{p_i^2}{q_i} \right) - 1 \right) = \frac{\chi^2_{GoF} \times E_{min}}{n \times (n - E_{min})}$$

For a Likelihood ratio:

$$E_L = -\frac{1}{\ln(q)} \times \sum_{i=1}^{k} \left(p_i \times \ln\left(\frac{p_i}{q_i}\right) \right) = -\frac{1}{\ln(q)} \times \frac{\chi^2_L}{2n}$$

Symbols used:

- q the minimum of all q-i's
- q_i the expected proportion in category i
- k the number of categories

3.2 Interpretation

For chi-square based test the interpretation is done by conversion to Cohen's w:

$$w = \sqrt{\frac{E_{\chi^2_{GoF}} \times (1-q)}{q}}$$

Then the interpretation table from Cohen's w can be used.

Cohen's classification for his w:

small:	w = .10,
medium:	w = .30,
large:	w = .50.

(Cohen, 1988, p. 227)

Proof

Cohen's w is defined as:

$$w = \sqrt{\frac{\chi^2_{GoF}}{n}}$$

$$\sqrt{\frac{E_{\chi^2_{GoF}} \times (1-q)}{q}} = \sqrt{\frac{\frac{\chi^2_{GoF} \times E_{min}}{n \times (n - E_{min})} \times (1-q)}{q}} = \sqrt{\frac{\chi^2_{GoF} \times E_{min} \times (1-q)}{n \times (n - E_{min}) \times q}}$$

$$= \sqrt{\frac{\chi^2_{GoF} \times E_{min} \times \left(1 - \frac{E_{min}}{n}\right)}{n \times (n - E_{min}) \times \frac{E_{min}}{n}}} = \sqrt{\frac{\frac{\chi^2_{GoF} \times E_{min} \times (n - E_{min})}{n}}{(n - E_{min}) \times E_{min}}} = \sqrt{\frac{\chi^2_{GoF}}{n}} = w$$

Q.E.D.

4 Sources

Johnston, Berry, and Mielke describe this E in *Measures of effect size for chi-squared and likelihood-ratio goodness-of-fit tests*.

$$E_{\chi^2} = \frac{q}{1-q}\left(\sum_{i=1}^{k}\frac{p_i^2}{q_i} - 1\right) \quad [5]$$

and

$$E_L = -\frac{1}{\ln(q)}\sum_{i=1}^{k} p_i \ln\left(\frac{p_i}{q_i}\right), \quad [6]$$

(Johnston et al., 2006, p. 413)

References

Cohen, J. (1988). *Statistical power analysis for the behavioral sciences* (2nd ed.). L. Erlbaum Associates.

Johnston, J. E., Berry, K. J., & Mielke, P. W. (2006). Measures of effect size for chi-squared and likelihood-ratio goodness-of-fit tests. *Perceptual and Motor Skills*, *103*(2), 412–414. https://doi.org/10.2466/pms.103.2.412-414